Behaviors of members predicted by a special case of a complex difference system model

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1 Introduction

There has been an increasing attention to the study of dynamical system theory in the social and behavioral sciences since the 1980s (Gregerson & Sailer, 1993; Heath, 2000), although the germ of the concept of dynamical system in social and personality psychology can be seen in the pioneering works of Lewin in the early 1930s (for example, Lewin, 1933).

Chino and Nakagawa (1983, 1990) have proposed DYNASCAL which identifies and depicts a set of estimated vector fields of members of a small group, given a set of longitudinal relational data matrices. This method is based on a *bifurcation model* of change in group structure over time, and assumes a set of general two-dimensional *nonlinear nonautonomous differential equations*,

$$\begin{pmatrix} dx_1/dt \\ dx_2/dt \end{pmatrix} = \begin{pmatrix} f_1(x_1, x_2, t) \\ f_2(x_1, x_2, t) \end{pmatrix}. \tag{1}$$

DYNASCAL considers the above nonautonomous system as a series of changes in an autonomous system over time, and fits the longitudinal relational data to this model. As a preprocessing process of these data, DYNASCAL applies MULTISCALE proposed by Ramsay (1977, 1982) to each of the longitudinal data matrices, and obtains configurations of members at each point in time, and then administers the Procrustes rotations to the neighboring pairs of configurations seriatim.

DYNASCAL utilizes qualitative theories of dynamical system, such as those of *singularities*, *structural stability*, and *bifurcations of vector field*. As a result, given the longitudinal relational data matrices among members, DYNASCAL draws a two-dimensional vector field on the estimated configuration of members at each time when observations are made. Furthermore, it depicts singularities and several fundamental solution curves peculiar to each of the vector fields. This enables interpretation of global and local dynamical properties of the group structure at each time.

On the one hand, DYNASCAL has several advantages over some traditional methods for analyzing group structures, i.e., sociograms and Markov process models for social networks (for example, Holland & Leinhardt, 1977). On the other hand, DYNASCAL has several disadvantages, too.

Firstly, it presupposes *asymmetric relationships* between members but the estimated relationships are symmetric. Secondly, it might not be fully justified mathematically to administer the Procrustes rotations to the neighboring pairs of configurations. Thirdly, DYNASCAL will not capture the so-called *chaotic behaviors* since it is restricted to a two-dimensional differential system. Fourthly, it is not possible for DYNASCAL to examine the behaviors of the system theoretically, since it merely estimates the solution curves using spline functions.

To overcome these difficulties, Chino (2002,2003a) has proposed some *complex diference system models* for social interaction. The most general form of these models is a *general nonlinear model* written as

$$\boldsymbol{z}_{j,n+1} = \boldsymbol{z}_{j,n} + \sum_{m=1}^{r} \sum_{k \neq j}^{N} \boldsymbol{D}_{jk,n}^{(m)} \boldsymbol{f}^{(m)}(\boldsymbol{z}_{k,n} - \boldsymbol{z}_{j,n}), \quad j = 1, 2, \cdots, N, \tag{2}$$

where,

$$\boldsymbol{f}^{(m)}(\boldsymbol{z}_{k,n} - \boldsymbol{z}_{j,n}) = \begin{pmatrix} \left(z_{k,n}^{(1)} - z_{j,n}^{(1)}\right)^m \\ \left(z_{k,n}^{(2)} - z_{j,n}^{(2)}\right)^m \\ \vdots \\ \left(z_{k,n}^{(p)} - z_{j,n}^{(p)}\right)^m \end{pmatrix}. \tag{3}$$

Moreover, $\boldsymbol{D}_{jk,n}^{(m)} = diag\left\{w_{jk,n}^{(1,m)}, \cdots, w_{jk,n}^{(p,m)}\right\}$, and

$$w_{jk,n}^{(l,m)} = a_n^{(l,m)} r_{j,n}^{(l,m)} r_{k,n}^{(l,m)} \sin\left(\theta_{k,n}^{(l,m)} - \theta_{j,n}^{(l,m)}\right), \; l = 1, 2, \cdots, p, \; m = 1, 2, \cdots, r. \tag{4}$$

It should be noticed that the state space of this model is not *real* but *complex*. Moreover, this model is composed of a set of *multivariate complex difference equations*. Our multivariate complex system models for social interaction may be naturally introduced applying the idea of HFM, which is a one-mode two-way asymmetric MDS (Chino & Shiraiwa, 1993), to longitudinal asymmetric relational data.

It is well known that difference equation models sometimes exhibit complicated chaotic behaviors even in the case of a simple *real* nonlinear equation such as, $x_{n+1} = (1+r)x_n - rx_n^2$, which is the famous *Verhulst process* (for example, Peitgen & Richter, 1986). It is also well known that even a simple *one-dimensional complex* difference system like Mandelbrot's difference equation, $z_{n+1} = z_n^2 + c$, can describe a variety of curious chaotic behaviors. Therefore, it is expected that our model can predict a variety of behaviors among members of a small group if it is appliable to real life situations. Chino (2003b) proposes some preliminary algorithms to fit a special case of this generalized model to the longitudinal relational data matrices.

Compared with data analytic models like DYNASCAL, theoretical models such as the Verhulst process and our difference model permit to examine their theoretical behaviors precisely. In fact, there has already been proposed a similar model of social systems, although it is restricted to two-person systems. Gregersen and Sailer (1993) examine a *metamodel* of two-person social systems described by the following *real* two-dimensional nonlinear difference equation,

$$x_{n+1} = r_x^1 x_n^2 + r_y^1 y_n^2 + r_{xy}^1 x_n y_n - u_x, \quad y_{n+1} = r_x^2 x_n^2 + r_y^2 y_n^2 + r_{xy}^2 x_n y_n - u_y, \tag{5}$$

and find curious chaotic behaviors. It is apparent that these equations include *Mandelbrot's Set* when $r_x^1 = 1$, $r_y^1 = -1$, $r_{xy}^2 = 2$, with the other rs equal to zero, as they note.

In the next section, we shall discuss some special cases of our general complex difference system model.

2 Some special cases of our general complex difference system model

Consider first a special case of our model described by eq. (2) through (4) when $p = 1$, $m = 2$, and $N = 2$. This is clearly a special two-person system. In this case, eq. (2) can be written as

$$z_{j,n+1} = a z_{jn}^2 + b z_{jn} + c, \tag{6}$$

where

$$a = w_{jk,n}^{(2)}, \quad b = 1 - w_{jk,n}^{(1)} - 2 w_{jk,n}^{(2)} z_{kn}, \quad c = w_{jk,n}^{(1)} z_{kn} + w_{jk,n}^{(2)} z_{kn}^2. \tag{7}$$

Now we shall make a strong assumption that the member j completely ignores the relationship with other member k. In other words, we shall assume that a, b, and c defined by equation (7) are all constants. If one notices that our model is a complex space model, it is evident that equation (6) is equivalent to the Mandelbrot's system.

In a somewhat more general case, when $p = 1$, $m = 2$ in the N-person system, eq. (2) can be written as the same as eq. (6), but

$$a = \sum_{k \neq j}^{N} w_{jk,n}^{(2)}, \quad b = 1 - \sum_{k \neq j}^{N} w_{jk,n}^{(1)} - 2 \sum_{k \neq j}^{N} w_{jk,n}^{(2)} z_{kn}, \tag{8}$$

and

$$c = \sum_{k \neq j}^{N} \left\{ w_{jk,n}^{(1)} z_{kn} + w_{jk,n}^{(2)} z_{kn}^2 \right\}. \tag{9}$$

In a more general case, when $p = 1$, eq. (2) can be written as follows,

$$z_{j,n+1} = a_r z_{jn}^r + a_{r-1} z_{jn}^{r-1} + \ldots + z_{jn} + a_0, \tag{10}$$

where

$$a_r = (-1)^r \left\{ \sum_{k \neq j}^{N} w_{jk,n}^{(r)} \right\}, \tag{11}$$

$$a_{r-1} = (-1)^{r-1} \left\{ \sum_{k \neq j}^{N} w_{jk,n}^{(r-1)} + \sum_{k \neq j}^{N} {}_{r}C_{r-1} w_{jk,n}^{(r)} z_{kn} \right\}, \tag{12}$$

$$a_{r-2} = (-1)^{r-2} \left\{ \sum_{k \neq j}^{N} w_{jk,n}^{(r-2)} + \sum_{k \neq j}^{N} {}_{r-1}C_{r-2} w_{jk,n}^{(r-1)} z_{kn} + \sum_{k \neq j}^{N} {}_{r}C_{r-2} w_{jk,n}^{(r)} z_{kn}^2 \right\}, \tag{13}$$

....,

and

$$a_0 = \sum_{k \neq j}^{N} \sum_{m=1}^{r} w_{jk,n}^{(m)} z_{kn}^m. \tag{14}$$

3 Discussion

These special cases of our general complex difference system model introduced above rest on the assumption that a certain member of a group suddenly begins to ignore all the sentiment relationships with the other members from a specified point in time. Such an assumption seems to be unusual, but it seems to be possible that we behave abnormally under some critical situations.

We will introduce some of the "unusual behaviors" of members in some special cases of our complex difference system model, and discuss the implications of these abnormal behaviors of members at the talk to be scheduled.

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